21102896

QP CODE: 21102896

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) EXAMINATIONS, OCTOBER 2021

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2019 Admission only

085B2F87

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the component equation and simplified component equation for the plane through $P_0(-3, 0, 7)$ perpendicular to $\mathbf{n} = 5\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.
2. State and prove the **Dot Product Rule** for differentiating vector functions.
3. Define the tangential and normal scalar components of acceleration.
4. What is k - component of curl of a vector ? .
5. Write the equation of the surface area differential for a parametrized surface S.
6. Describe the one sided surface of Mobius band.
7. Check whether the following set S of integers constitute a complete set of residues modulo 7 or not:
   $S = \{-12, -4, 11, 13, 22, 82, 91\}$.
8. Define *pseudoprime*.
9. Prove: $\phi(3n) = 3\phi(n)$ if and only if $3|n$.
10. Using definition of Laplace Transform, prove that $\mathscr{L}(\sin \omega t) = \frac{\omega}{s^2+\omega^2}$.
11. Find $\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+\omega^2)}\right\}$.

12. Define the convolution of the functions $f(t)$ and $g(t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Give the standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$. Also write two different parametric equations for the line through $P(-3, 2, -3)$ and $Q(1, -1, 4)$.

14. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

15. Find the line integral of $f(x, y, z) = 2x + 3y + z$ over the straight line segment from $(0, 1, -1)$ to $(1, 1, 2)$.

16. Evaluate $\int(F.\,dr)$, where $F(x, y, z) = zi + xyj - y^2k$ along the curve $C; r(t) = t^2i + tj + \sqrt{t}k, 0 \leq t \leq 1$.

17. a) What is the component test for the exactness of the differential form $Mdx + Ndy + Pdz$.
    b) Is the field $F = (z + y)i + zj + (y + x)k$ conservative? Why? .

18. Prove: If p and q are distinct primes with $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then $a^{pq} \equiv a \pmod{pq}$.

19. Derive the congruence: $a^7 \equiv a \pmod{42}$ for all a.

20. Let $f(t)$, $f'(t)$ be continuous and satisfy the growth restriction for all $t \geq 0$. Also let $f''(t)$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Prove that the Laplace transform of $f''(t)$ satisfies $\mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0)$.

21. Solve $y'' - \frac{1}{4}y = 0,\ y(0) = 4,\ y'(0) = 0$ using Laplace Transform.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.

1. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.
2. Find the **curvature** for the helix $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + bt\mathbf{k}, a, b \geq 0, a^2 + b^2 \neq 0.$

23. Verify any one form of Green's Theorem for the vector field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r(t) = (cost)i + (sint)j$,
$0 \leq t \leq 2\pi$.

24.
1. State and prove Wilson's theorem.
2. Prove the converse of Wilson's theorem.

25.
1. State and prove Convolution theorem.
2. Solve $y(t) - \int_0^t y(\tau) \sin(t - \tau)\, d\tau = \cos t$.

(2×15=30)